Math 270: Differential Equations Day 12 Part 1

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited Part 2

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Recall: The Method of Undetermined Coefficients

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{rt},$$

where m is a nonnegative integer, use the form

(14) $$y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{rt},$$

with

(i) $s = 0$ if r is not a root of the associated auxiliary equation;
(ii) $s = 1$ if r is a simple root of the associated auxiliary equation; and
(iii) $s = 2$ if r is a double root of the associated auxiliary equation.

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Recall: The Method of Undetermined Coefficients

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t}\cos\beta t \\ \text{or} \\ Ct^m e^{\alpha t}\sin\beta t \end{cases}$$

for $\beta \neq 0$, use the form

$$y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t}\cos\beta t + t^s(B_m t^m + \cdots + B_1 t + B_0)e^{\alpha t}\sin\beta t\,, \tag{15}$$

with

(iv) $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and
(v) $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Recall: The Superposition Principle

Superposition Principle

Theorem 3. If y_1 is a solution to the differential equation

$$ay'' + by' + cy = f_1(t) ,$$

and y_2 is a solution to

$$ay'' + by' + cy = f_2(t) ,$$

then for any constants k_1 and k_2, the function $k_1y_1 + k_2y_2$ is a solution to the differential equation

$$ay'' + by' + cy = k_1f_1(t) + k_2f_2(t) .$$

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Method of Undetermined Coefficients (Revisited)

To find a particular solution to the differential equation

$$ay'' + by' + cy = P_m(t)e^{rt},$$

where $P_m(t)$ is a polynomial of degree m, use the form

(13) $$y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{rt};$$

if r is not a root of the associated auxiliary equation, take $s = 0$; if r is a simple root of the associated auxiliary equation, take $s = 1$; and if r is a double root of the associated auxiliary equation, take $s = 2$.

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Method of Undetermined Coefficients (Revisited)

To find a particular solution to the differential equation

$$ay'' + by' + cy = P_m(t)e^{\alpha t}\cos\beta t + Q_n(t)e^{\alpha t}\sin\beta t\,, \quad \beta \neq 0\,,$$

where $P_m(t)$ is a polynomial of degree m and $Q_n(t)$ is a polynomial of degree n, use the form

$$\text{(14)} \qquad y_p(t) = t^s(A_k t^k + \cdots + A_1 t + A_0)e^{\alpha t}\cos\beta t + t^s(B_k t^k + \cdots + B_1 t + B_0)e^{\alpha t}\sin\beta t\,,$$

where k is the larger of m and n. If $\alpha + i\beta$ is not a root of the associated auxiliary equation, take $s = 0$; if $\alpha + i\beta$ is a root of the associated auxiliary equation, take $s = 1$.

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Ex 4.5: Write down the form of a particular solution to $y'' - 5y' + 6y = \left(8t^3 + 3t - 5\right)e^{2t}$

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Example 5 Write down the form of a particular solution to the equation

$$y'' + 2y' + 2y = 5e^{-t}\sin t + 5t^3e^{-t}\cos t\,.$$

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 2)

Example 6 Write down the form of a particular solution to the equation

$$y''' + 2y'' + y' = 5e^{-t}\sin t + 3 + 7te^{-t}.$$